```
> restart:with(plots):with(student):
Warning, the name changecoords has been redefined
```

as students ask me questions about the sample exam, I will post the solutions into this file.

```
> #page 2 III #3
```

evalaute in rectangular first

```
> Int(Int(Int( 1, z=0 .. sqrt(1-x^2-y^2)),x=y..sqrt(1-y^2)),y=0..
  sqrt(2)/2)=int(int(int( 1, z=0 ..
  sqrt(1-x^2-y^2)),x=y..sqrt(1-y^2)),y=0.. sqrt(2)/2);
Warning, unable to determine if -(1-y^2)^(1/2) is between y and (1-y^2)^(1/2); try
to use assumptions or set _EnvAllSolutions to true
```

$$\int_0^{\frac{\sqrt{2}}{2}} \int_y^{\sqrt{1-y^2}} \int_0^{\sqrt{1-x^2-y^2}} 1 \, dz \, dx \, dy = \frac{\pi}{12}$$

evalaute in sphereical as requested

```
> Int(Int(Int( rho^2*sin(phi), rho=0..1),phi=0..Pi/2),theta=0..
  Pi/4)=int(int(int( rho^2*sin(phi),
  rho=0..1),phi=0..Pi/2),theta=0.. Pi/4);
```

$$\int_0^{\frac{\pi}{4}} \int_0^{\frac{\pi}{2}} \int_0^1 \rho^2 \sin(\phi) \, d\rho \, d\phi \, d\theta = \frac{\pi}{12}$$

```
> Int(Int(Int( r, z=0..sqrt(1-r^2)),r=0..1),theta=0..
  Pi/4)=int(int(int( r, z=0..sqrt(1-r^2)),r=0..1),theta=0.. Pi/4);
```

$$\int_0^{\frac{\pi}{4}} \int_0^1 \int_0^{\sqrt{1-r^2}} r \, dz \, dr \, d\theta = \frac{\pi}{12}$$

```
> #page 1 II #8
> # as a path integral over 3 line segments
> with(VectorCalculus):SetCoordinates( 'cartesian'[x,y] );
> LineInt( VectorField( <-y*exp(x),x*exp(y)> ), LineSegments( <0,0>,
  <1,0>, <1,1>,<0,0> ),inert)=LineInt( VectorField(
  <-y*exp(x),x*exp(y)> ), LineSegments( <0,0>, <1,0>, <1,1>
  ,<0,0>));# closed curve
```

[*&x*, *, +, -, ., <,>, <|>, *AddCoordinates*, *ArcLength*, *BasisFormat*, *Binormal*, *CrossProd*, *CrossProduct*, *Curl*, *Curvature*, D, *Del*, *DirectionalDiff*, *Divergence*, *DotProd*, *DotProduct*, *Flux*, *GetCoordinateParameters*, *GetCoordinates*, *Gradient*, *Hessian*, *Jacobian*, *Laplacian*, *LineInt*, *MapToBasis*, *Nabla*, *Norm*, *Normalize*, *PathInt*, *PrincipalNormal*, *RadiusOfCurvature*, *ScalarPotential*, *SetCoordinateParameters*, *SetCoordinates*, *SurfaceInt*, *TNBFrame*, *Tangent*, *TangentLine*, *TangentPlane*, *TangentVector*, *Torsion*, *Vector*, *VectorField*, *VectorPotential*, *Wronskian*, *diff*, *evalVF*, *int*, *limit*, *series*]

$$cartesian_{x,y}$$

$$2\int_0^1 0\,dt+\int_0^1 \mathbf{e}^t\,dt=-1+\mathbf{e}$$

```
> Int(Int( Diff(x*exp(y),x)-Diff(-y*exp(x),y), y=0 .. x),x=0 ..
  1)=int(int( diff(x*exp(y),x)-diff(-y*exp(x),y), y=0 .. x),x=0 ..
  1);
```

$$\int_0^1\int_0^x\left(\frac{\partial}{\partial x}(x\,\mathbf{e}^y)\right)-\left(\frac{\partial}{\partial y}(-y\,\mathbf{e}^x)\right)dy\,dx=-1+\mathbf{e}$$

```
> #page 2 III #10
> SetCoordinates( 'cartesian'[x,y,z] );
```

$$cartesian_{x,y,z}$$

```
> F := VectorField( <y*sin(z),x*sin(z),x*y*cos(z)> );
```

$$F:=y\sin(z)\,\underline{\mathbf{e}}_{\mathbf{x}}+x\sin(z)\,\underline{\mathbf{e}}_{\mathbf{y}}+x\,y\cos(z)\,\underline{\mathbf{e}}_{\mathbf{z}}$$

```
> Divergence( F );Curl( F );
```

$$-x\,y\sin(z)$$

$$0\,\underline{\mathbf{e}}_{\mathbf{x}}$$

```
> # page 2 III #9 a and b
> SetCoordinates( 'cartesian'[x,y,z] ):v := VectorField( <yz,xz,xy>
  );
```

$$v:=yz\,\underline{\mathbf{e}}_{\mathbf{x}}+xz\,\underline{\mathbf{e}}_{\mathbf{y}}+xy\,\underline{\mathbf{e}}_{\mathbf{z}}$$

```
> ScalarPotential( v );
```

$$yz\,x+xz\,y+xy\,z$$

```
> SetCoordinates( 'cartesian'[x,y,z] ):v := VectorField(
  <(sec(x))^2,z,y+2*z> );
> ScalarPotential( v );
```

$$v:=\sec(x)^2\,\underline{\mathbf{e}}_{\mathbf{x}}+z\,\underline{\mathbf{e}}_{\mathbf{y}}+(y+2\,z)\,\underline{\mathbf{e}}_{\mathbf{z}}$$

$$\frac{\sin(x)}{\cos(x)}+z\,y+z^2$$

```
>
>
```